Audubon Public Schools
Engaging Students ~ Fostering Achievement ~ Cultivating 21st Century Global Skills
Written By: Patricia Martel
Revised By: Ron Latham
Approved June 2017

Course Title: Calculus **Unit Name: Prerequisites: Slope, Functions, Logarithm & Trigonometry** **Grade Level: 12**

Content Statements This unit reviews in brief the essential concepts of algebra and pre-calculus.	**NJSLS** N-RN.1-3 N-CN.7-9 A-SSE.3 F-LE.1-5 F-TF.1-5
Overarching Essential Questions How do we find solutions to equations? How do we graph equations? What are the 10 Basic functions and their distinguishing characteristics?	**Overarching Enduring Understandings** There are several functions specific methods of solution but there are several methods that can be universally applied to solve problems.
Unit Essential Questions How can the slope of a line be interpreted? How is the equation for a line determined from two points? What is the significance of a function's domain and range? What is a piecewise function and how are they graphed? What types of phenomena are modeled by exponential equations? How are exponential and logarithmic equations simplified and solved? How do parametric equations simplify modeling for motion problems? What are the properties of sinusoidal functions? How can trig values and inverse trig values be found using technology and the unit circle?	**Unit Enduring Understandings** The linear equations y= m x +b can be used to model a variety of phenomena, and the slope of a tangent line bears an important relationship to the curves of a variety of functions.
Unit Rationale This unit helps to identify the areas of strength and weakness in this course and reminds and remediates the essential skills.	**Unit Overview** The study of calculus begins with the study of functions, and students must be well-versed in the basics of lines, slope, and graphing.
Key Terms Factor – set of terms, monomial, binomial that when multiplied together equal the original expression Function- a set of points or equation were each x coordinate is unique Root- the solution to one of the factors of a function Solution- the result of setting a factor or root equal to zero and solving Zero- the x coordinate were a function crosses the x-axis, making the y value zero	
Resources Calculus: Graphical, Numerical, Algebraic, Finney, Demana, Waits, and Kennedy, Prentice-Hall, ISBN 0-13-063131-0 TI 83 or 84 Graphing Calculator Required TI website for reference materials	
Suggested Activities for Inclusion in Lesson Planning Graphing lines by hand	

Using the graphing utilities to find trend lines and lines of best fit for data Determining domain, range, and other properties of a function given a graph or an equation Graphing piecewise functions and determining the equation for the function bases on the graph Setting up and solving exponential functions Solving problems with half life, radioactive decay, and population growth. Setting up and solving parametric motion problems Analyzing the graphs of sine and cosine functions Graphing trigonometric functions Modeling periodic phenomena using trigonometry Finding values using the unit circle

NOTE: This unit provides an opportunity to determine the overall skill level of the class. Use it to review core concepts like factoring and simplifying rational expressions, evaluating logarithms, and solving right triangles if the class is weak in those topics. A global note: In keeping with the suggestions of groups like the College Board and the NCTM, you should teach this course with graphing calculators integrated into the lessons. That means students should explore limits by hand and using the calculators, and that they should learn how to use the derivative and integral functions on the calculator. This doesn't mean we don't teach by hand techniques (we must!), but it is up to the teacher to find a balance. We should also use the calculator as an aide to teaching theory: that the derivative is a slope, area an integral, and so on.

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Course Title: Calculus Unit Name: Limits Grade Level: 12

Content Statements This unit introduces the analytical concept of a limit.	**NJSLS** A-APR.6 A-REI.4
Overarching Essential Questions What is a limit? What do we use limits for?	**Overarching Enduring Understandings** The limit is the y value at a given value of x. If the point does not exit we look to predict its value base on the values before and after it.
Unit Essential Questions How is the limit of function different from its value? How can limits be found algebraically, graphically, and using tables? What are left and right-handed limits? How can the fact that the limit as x approaches zero of (sin x)/x be used to find other limits? What are limits at infinity and how are they related to horizontal asymptotes? What is the value of an end behavior model for understanding the behavior of a function? What are the different types of function discontinuity? How is the average rate of change of a function related to the slope? How can limits be used to find the instantaneous rate of change of a function at a point?	**Unit Enduring Understandings** Limits allow mathematicians to work with numbers so small that they can be assumed to be nearly zero. Limits allow us to analyze functions and talk about the rate of change at an exact point, instead of needing two points to take an average.

Unit Rationale	Unit Overview
Limits are used to approximate undefined quantities, a necessary skill for deal with derivatives and later, integrals.	The limit is a fundamental concept in higher math. A theoretical understanding of the limit allows us to work with infinitesimally small values, building the bridge from estimated slopes and areas to the exact values found by applying derivatives and integrals.

Key Terms
Asymptote – invisible line that a function either “hugs” or oscillates around
Continuity – a function or portion of a function without holes or skips, that can be drawn with one pen stroke
Discontinuous – a function or portion of a function with a hole and skip forcing additional pen strokes to be written
Infinity- having no end or no beginning
Instantaneous – at an exact moment
Limits – the approximation of a y value at a specific x value or as x approaches an infinity
Oscillating- fluctuating around a value, higher then lower almost too fast to be seen, appears almost fractal as you zoom in

Resources
Calculus: Graphical, Numerical, Algebraic, Finney, Demana, Waits, and Kennedy, Prentice-Hall, ISBN 0-13-063131-0
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Suggested Activities for Inclusion in Lesson Planning
Graphing functions, estimating slopes, and analyzing behavior at given points.
Creating tables of values for functions and using the tables to find slopes on small intervals
Locating points of discontinuity and differentiating limits from values on discontinuous graphs
Finding the limit of sin x / x at 0, then developing the algebraic steps for applying this limit to other problems
Calculating the average velocity of a falling object.
Finding horizontal asymptotes and limits at infinity for functions.
Ordering or ranking families of functions based on their rate of growth at infinity and negative infinity
Reducing rational functions to find limits and recognizing the domain restrictions on a reduced rational function.

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Course Title: Calculus **Unit Name: Derivatives** **Grade Level: 12**

Content Statements	NJSLS
This unit introduces derivatives and their calculation through limits and algebraic manipulation.	A-REI.4
Overarching Essential Questions What is a derivative? How can derivatives be calculated use derivative rules?	**Overarching Enduring Understandings** Derivatives tell us how quickly or slowly the slope of a line is changing?
Unit Essential Questions What is the formal definition of a derivative? How can derivatives of polynomial functions be calculated using the formal definition? What can you learn about the shape of a graph by finding it’s derivative at a point? When is a function differentiable? What are the power, product, quotient, and chain rules, and how can they be applied to find derivatives?	**Unit Enduring Understandings** The derivative is the slope of a line tangent to a graph at a point. It can be used to represent instantaneous velocity, and it can be calculated using a variety of techniques or rules.

How is the derivative related to velocity and acceleration? What are the derivatives of the trigonometric and logarithmic functions? What is implicit differentiation?	
Unit Rationale Derivatives are essential skills to any calculus course.	**Unit Overview** The study of calculus begins with the study of functions, and students must be well-versed in the basics of lines, slope, and graphing.

Key Terms Acceleration- the rate at which the velocity of an object is changing Continuity- being in one piece with no breaks or skips Derivative- the melting down of a function to its most basic elements, the rate of change of a function Differentiation- changing a function to its derivative Implicit Differentiation- finding a derivative with 2 variables without solving for one first Velocity- the rate at which an object is changing
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Suggested Activities for Inclusion in Lesson Planning Finding equations for derivatives using power, product, chain, quotient, and other rules Estimating the derivative by finding the slope of a secant line Determining the differentiability of a function based on its continuity Finding velocity and acceleration given a displacement equation. Finding maximum heights and maximum velocity using calculus techniques Applying implicit differentiation to find the derivative of a function with multiple variables. Finding the slope of a line tangent to circle using implicit differentiation. Analyzing trip and exponential functions using calculus Using and understanding of Physics students will calculate the velocity of an object using a CBL

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Course Title: Calculus **Unit Name: Applications of Derivatives** **Grade Level: 12**

Content Statements This unit contains many scenarios and applications for derivatives.	**NJSLS** A-REI.10-12
Overarching Essential Questions What are derivatives used for?	**Overarching Enduring Understandings** Derivatives reduce the degree of a function. From volume we derive area, from distance we derive velocity etc. As a result, there are countless applications of the derivative.
Unit Essential Questions What are extrema and how can the derivative be used to find them? What is concavity and how is it related to the second derivative?	**Unit Enduring Understandings** Because the derivative is a rate of change, it can be applied to a variety of problems in which properties are increasing and decreasing. The derivative is also useful in finding maxima and minima, making it an important tool in optimization.

What are the implications of the Mean Value Theorem? How can the first and second derivatives be used as aides in graphing functions? How can the derivative be applied to optimization problems? What are related rates problems? How can derivatives be used to solve multi-variable problems where several variables change with respect to time?	
Unit Rationale It is always important to learn applications for the concepts we learn.	**Unit Overview** Derivatives have a variety of applications, from studying the nature of curves themselves to determining the rates of change of real world phenomena. Optimization and related rate problems are major engineering and business applications of derivatives.
Key Terms Concavity – curved inward Inflection – point of changing concavity Maxima – highest or largest value Minima – lowest or smallest value Optimization – finding a max or following value to make decisions in business etc.	
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Suggested Activities for Inclusion in Lesson Planning Graphing functions to find maxima, minima, zeros, and other behavior Calculating second derivatives Determining concavity from equations and graphs, and by using the second derivative test Finding, interpreting inflection points Modeling optimization problems using algebra and geometry Using calculus to find optimal values in modeling problems. Calculate regression models for real world applications and derive their slope equations. Use the result to optimize equations.	

NOTE: The teacher should try to find a balance between by-hand techniques and calculator techniques. As a rule, problems with low-order polynomials, basic exponential functions, and basic trig should be solved by hand. For optimization and related rates problems, interpreting and setting up the equation should be the primary focus of the lesson, and using the calculator can keep the “solving” part of the problem from becoming prohibitively difficult.

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Course Title: Calculus Unit Name: Integrals & the Fundamental Theorem of Calculus
Grade Level: 12

Content Statements This unit introduces integration	**NJSLS** A-CED.3 A-REI.7 A-REI.10-12
Overarching Essential Questions What is integration? What is the definite integral?	**Overarching Enduring Understandings** An integral is the opposite of a derivative, i.e. adding A degree with each usage.
Unit Essential Questions How is the area beneath a curve related to the displacement of an object in motion? How can Rectangular Approximation methods be used to estimate areas? What is a definite integral? What geometric methods can be used to calculate integrals? How can the trapezoid rule be used to estimate areas? How is the anti-derivative related to the definite and indefinite integral?	**Unit Enduring Understandings** The “definite integral” is the area beneath the curve. It can sometimes be found using geometry, but the anti-derivative can also be applied to finding integrals. The Fundamental Theorem of Calculus states that the anti-derivative is the indefinite integral of a function.
Unit Rationale Integrals are an essential element of any college Calculus course.	**Unit Overview** The core concept of calculus is that the anti-derivative is equal to the indefinite integral, linking areas and slopes into one topic and consolidating the study of graphs and their motion.

Key Terms Anti-derivatives – the opposite predictive method of integration Area Under a curve – the two dimensional value of space under a curve Definite Integral – the value of an integral over a specified interval Indefinite Integral – the value of an integral over a general interval Integral – the opposite of a derivative Overestimate – a value intentionally constructed to arrive at a larger value than necessary Underestimate – a value intentionally constructed to arrive at a smaller value than necessary
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Suggested Activities for Inclusion in Lesson Planning Finding the area beneath a curve using geometric formulas Using the TI-83 to find the values for finding area using Rectangular Approximation Determining whether MRAM, LRAM, or RRAM will yield over or underestimates. Applying the trapezoid rule to find areas of irregular shapes Exploring the relationship between anti-derivatives and integrals graphically and algebraically. Solving definite and indefinite integrals using the power rule and the substitution principle. Applying the Fundamental Theorem of Calculus to find definite integrals Create models of an area under a curve to illustrate LRAM, RRAM, MRAM & Trapezoidal rule and to illustrate how they work. Build models of shells and washers using foam board to model how the calculus “constructs” each solid

NOTE: Take the extra time here to emphasize that the integral is an area. Have the students solve problems in integral notation using geometric methods: for example, the areas of triangular, trapezoidal, and circular regions. Otherwise the power rule for integrals becomes a “trick” that students cannot apply outside of integral notation. Do not be shy about teaching calculator methods of integration, but do it after the students have been assessed once or twice on by-hand techniques.

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Course Title: Calculus **Unit Name: Applications of Integrals** **Grade Level: 12**

Content Statements This unit covers many real world applications of the definite integral.	**NJSLS** A-CED.3 A-REI.7 A-REI.10-12
Overarching Essential Questions Why do we learn to integrate?	**Overarching Enduring Understandings** Integration is essential to solving many problems more simply and elegantly than algebra will allow.
Unit Essential Questions How can integrals be used to find the area between two curves? How can the disk and shell methods be used to find the volumes and densities of solids? What is the relationship between the definite integral and the accumulation of a quantity? What is Hooke's Law? How can integrals be applied to the study of forces, work, and energy? What is the formula for the length of a curve?	**Unit Enduring Understandings** Integrals have a variety of applications. They can be used to find area, volume, density, the force acting upon a spring, the length of a curve, and other physical properties. Because an integral is a sum, it can solve accumulation problems.
Unit Rationale It is always important to use direct applications of the skills we learn, especially when these new skills make it easier or faster to solve problems.	**Unit Overview** Study of the physical applications of integrals, including volumes of solids and Work-Energy problems, leads directly to the engineering sciences.

Key Terms
Disk- volume of a three dimensional solid with a radius perpendicular to the axis of rotation
Shell- volume of a three dimensional solid with a radius parallel to the axis of rotation
Washer- volume of a three dimensional solid with and inner hollow space

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Suggested Activities for Inclusion in Lesson Planning
Graphing two curves on the TI-83 and finding points of intersection
Exploring the differences between distance and displacement, speed and velocity
Applying geometric principles to extend the concept of area to volume and density
Modeling solids using disk and shell methods, then finding the Volumes of those solids using definite integrals
Solving accumulation problems and determining the constant of integrations "c" for specific models.
Exploring spring tension using weights
Applying Hooke's law to find the work and energy in spring problems.
Modeling and solving distance-work problems involving anchors-and-chains, rockets, and other variable-weight objects
Applying the arc length formula to find lengths of curves.
Using foam board, build models of shell, disk and washer approximations of volume.
Take a cross-sectional photo of a Bundt cake or donut, find equations to model to space

Appendix

Differentiation	
Enrichment	• Utilize collaborative media tools • Provide differentiated feedback • Opportunities for reflection • Encourage student voice and input • Model close reading • Distinguish long term and short term goals
Intervention & Modification	• Utilize “skeleton notes” where some required information is already filled in for the student • Provide access to a variety of tools for responses • Provide opportunities to build familiarity and to practice with multiple media tools • Leveled text and activities that adapt as students build skills • Provide multiple means of action and expression • Consider learning styles and interests • Provide differentiated mentors • Graphic organizers
ELLs	• Pre-teach new vocabulary and meaning of symbols • Embed glossaries or definitions • Provide translations • Connect new vocabulary to background knowledge • Provide flash cards • Incorporate as many learning senses as possible • Portray structure, relationships, and associations through concept webs • Graphic organizers

21st Century Skills

- Creativity
- Innovation
- Critical Thinking
- Problem Solving
- Communication
- Collaboration

Integrating Technology

- Chromebooks
- Internet research
- Online programs
- Virtual collaboration and projects
- Presentations using presentation hardware and software